

Sage Quick Reference (Basic Math)

Peter Jipsen, version 1.1 (w/modification by nu)

latest version at wiki.sagemath.org/quickref

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Aim: map standard math notation to Sage commands

Notebook(とコマンドライン) Notebook (and commandline)

セルの評価: `<shift-enter>`

`com(tab)` *command* と補完しようとする.

`command?(tab)` ドキュメントを表示

`command??(tab)` ソースを表示

`a.(tab)` オブジェクト *a* のメソッドを表示 (more: `dir(a)`)

`search_doc('string or regexp')` ドキュメントへのリンク

`search_src('string or regexp')` ソースへのリンクを表示

`lprint()` L^AT_EX 形式の出力に切替える

`version()` Sage のバージョンを表示

Insert cell: セルの間の青い線をクリック

Delete cell: 内容を消してから backspace

Evaluate cell: `<shift-enter>`

`com(tab)` tries to complete *command*

`command?(tab)` shows documentation

`command??(tab)` shows source

`a.(tab)` shows all methods for object *a* (more: `dir(a)`)

`search_doc('string or regexp')` shows links to docs

`search_src('string or regexp')` shows links to source

`lprint()` toggle L^AT_EX output mode

`version()` print version of Sage

Insert cell: click on blue line between cells

Delete cell: delete content then backspace

数の型 Numerical types

整数: $\mathbb{Z} = \mathbb{ZZ}$ e.g. `-2 -1 0 1 10^100`

有理数: $\mathbb{Q} = \mathbb{QQ}$ e.g. `1/2 1/1000 314/100 -42`

小数: $\mathbb{R} \approx \mathbb{RR}$ e.g. `.5 0.001 3.14 -42`

複素数: $\mathbb{C} \approx \mathbb{CC}$ e.g. `1+i 2.5-3*i`

Integers: $\mathbb{Z} = \mathbb{ZZ}$ e.g. `-2 -1 0 1 10^100`

Rationals: $\mathbb{Q} = \mathbb{QQ}$ e.g. `1/2 1/1000 314/100 -42`

Decimals: $\mathbb{R} \approx \mathbb{RR}$ e.g. `.5 0.001 3.14 -42`

Complex: $\mathbb{C} \approx \mathbb{CC}$ e.g. `1+i 2.5-3*i`

基本的な定数と関数 Basic constants and functions

定数: $\pi = \text{pi}$ $e = \text{e}$ $i = \text{i}$ $\infty = \text{oo}$

近似値: `pi.n(digits=18) = 3.14159265358979324`

関数: `sin cos tan sec csc cot sinh cosh tanh sech csch`

`coth log ln exp`

$ab = \text{a*b}$ $\frac{a}{b} = \text{a/b}$ $a^b = \text{a^b}$ $\sqrt{x} = \text{sqrt}(x)$

$\sqrt[n]{x} = \text{x}^(1/n)$ $|x| = \text{abs}(x)$ $\log_b(x) = \text{log}(x,b)$

不定元: e.g. `t,u,v,y = var('t u v y')`

函数定義: e.g. $f(x) = x^2$

(微分等ができるシンボリックな) 函数として: `f(x)=x^2`

Python 関数として定義する: `f=lambda x: x^2` または

`def f(x): return x^2`

Constants: $\pi = \text{pi}$ $e = \text{e}$ $i = \text{i}$ $\infty = \text{oo}$

Approximate: `pi.n(digits=18) = 3.14159265358979324`

Functions: `sin cos tan sec csc cot sinh cosh tanh sech csch`

`coth log ln exp`

$ab = \text{a*b}$ $\frac{a}{b} = \text{a/b}$ $a^b = \text{a^b}$ $\sqrt{x} = \text{sqrt}(x)$

$\sqrt[n]{x} = \text{x}^(1/n)$ $|x| = \text{abs}(x)$ $\log_b(x) = \text{log}(x,b)$

Symbolic variables: e.g. `t,u,v,y = var('t u v y')`

Define function: e.g. $f(x) = x^2$

As symbolic function (can integrate, etc): `f(x)=x^2` or

As Python function: `f=lambda x: x^2` or

`def f(x): return x^2`

式に対する操作 Operations on expressions

`factor(...)` `expand(...)` `(...).simplify...`

シンボリックな等式: `f(x)==g(x)`

`_` は直前の出力

`+_a -_a *_a /_a` で等式を操作できる

$f(x) = g(x)$ を解く: `solve(f(x)==g(x),x)`

`solve([f(x,y)==0, g(x,y)==0], x,y)`

$x \in [a, b]$ s.t. $f(x) \approx 0$ を探す: `find_root(f(x), a, b)`

$\sum_{i=k}^n f(i) = \text{sum}([f(i) \text{ for } i \text{ in } [k..n]])$

$\prod_{i=k}^n f(i) = \text{prod}([f(i) \text{ for } i \text{ in } [k..n]])$

`factor(...)` `expand(...)` `(...).simplify...`

Symbolic equations: `f(x)==g(x)`

`_` is previous output

`+_a -_a *_a /_a` manipulates equation

Solve $f(x) = g(x)$: `solve(f(x)==g(x),x)`

`solve([f(x,y)==0, g(x,y)==0], x,y)`

`find_root(f(x), a, b)` find $x \in [a, b]$ s.t. $f(x) \approx 0$

$\sum_{i=k}^n f(i) = \text{sum}([f(i) \text{ for } i \text{ in } [k..n]])$

$\prod_{i=k}^n f(i) = \text{prod}([f(i) \text{ for } i \text{ in } [k..n]])$

微分積分 Calculus

$\lim_{x \rightarrow a} f(x) = \text{limit}(f(x), x=a)$

$\lim_{x \rightarrow a^-} f(x) = \text{limit}(f(x), x=a, \text{dir}='minus')$

$\lim_{x \rightarrow a^+} f(x) = \text{limit}(f(x), x=a, \text{dir}='plus')$

$\frac{d}{dx} f(x) = \text{diff}(f(x), x)$

$\frac{\partial}{\partial x} f(x, y) = \text{diff}(f(x, y), x)$

`diff = differentiate = derivative`

$\int f(x) dx = \text{integral}(f(x), x)$

`integral = integrate`

$\int_a^b f(x) dx = \text{integral}(f(x), x, a, b)$

次数 n の a に関する Taylor 多項式: `taylor(f(x), x, a, n)`

$\lim_{x \rightarrow a} f(x) = \text{limit}(f(x), x=a)$

$\lim_{x \rightarrow a^-} f(x) = \text{limit}(f(x), x=a, \text{dir}='minus')$

$\lim_{x \rightarrow a^+} f(x) = \text{limit}(f(x), x=a, \text{dir}='plus')$

$\frac{d}{dx} f(x) = \text{diff}(f(x), x)$

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`diff = differentiate = derivative`

$\int f(x) dx = \text{integral}(f(x), x)$

`integral = integrate`

$\int_a^b f(x) dx = \text{integral}(f(x), x, a, b)$

Taylor polynomial, deg n about a : `taylor(f(x), x, a, n)`

二次元グラフィックス 2d graphics

`line([(x1, y1), ..., (xn, yn)], options)`

`polygon([(x1, y1), ..., (xn, yn)], options)`

`circle((x, y), r, options)`

`text("txt", (x, y), options)`

options は `plot.options` にあるものを使用,

例 `thickness=pixel, rgbcolor=(r, g, b), hue=h`

(ただし $0 \leq r, b, g, h \leq 1$)

縦横比の調整には `figsize=[w, h]`

`plot(f(x), xmin, xmax, options)`

`parametric_plot((f(t), g(t)), tmin, tmax, options)`

`polar_plot(f(t), tmin, tmax, options)`

グラフの結合: `circle((1, 1), 1)+line([(0, 0), (2, 2)])`

`animate(list of graphics objects, options).show(delay=20)`

`line([(x1, y1), ..., (xn, yn)], options)`

`polygon([(x1, y1), ..., (xn, yn)], options)`

`circle((x, y), r, options)`

`text("txt", (x, y), options)`

options as in `plot.options`,

e.g. `thickness=pixel, rgbcolor=(r, g, b), hue=h,`

where $0 \leq r, b, g, h \leq 1$

use option `figsize=[w, h]` to adjust aspect ratio

`plot(f(x), xmin, xmax, options)`

`parametric_plot((f(t), g(t)), tmin, tmax, options)`

`polar_plot(f(t), tmin, tmax, options)`

combine graphs: `circle((1, 1), 1)+line([(0, 0), (2, 2)])`

`animate(list of graphics objects, options).show(delay=20)`

三次元グラフィックス 3d graphics

`line3d([(x1, y1, z1), ..., (xn, yn, zn)], options)`

`sphere((x, y, z), r, options)`

`tetrahedron((x, y, z), size, options)`

`cube((x, y, z), size, options)`

`octahedron((x,y,z),size,options)`
`dodecahedron((x,y,z),size,options)`
`icosahedron((x,y,z),size,options)`
options の例 `aspect_ratio=[1,1,1]` `color='red'` `opacity`
`plot3d(f(x,y),[x_b,x_e],[y_b,y_e],options)`
 オプションに `plot_points=[m,n]` or `plot3d_adaptive` を使う
`parametric_plot3d((f(t),g(t),h(t)),[t_b,t_e],options)`
`parametric_plot3d((f(u,v),g(u,v),h(u,v)),`
 `[u_b,u_e],[v_b,v_e],options)`
 graphics objects を結合するには + を使う
`line3d([(x1,y1,z1),...,(xn,yn,zn)],options)`
`sphere((x,y,z),r,options)`
`tetrahedron((x,y,z),size,options)`
`cube((x,y,z),size,options)`
`octahedron((x,y,z),size,options)`
`dodecahedron((x,y,z),size,options)`
`icosahedron((x,y,z),size,options)`
options e.g. `aspect_ratio=[1,1,1]` `color='red'` `opacity`
`plot3d(f(x,y),[x_b,x_e],[y_b,y_e],options)`
 add option `plot_points=[m,n]` or use `plot3d_adaptive`
`parametric_plot3d((f(t),g(t),h(t)),[t_b,t_e],options)`
`parametric_plot3d((f(u,v),g(u,v),h(u,v)),`
 `[u_b,u_e],[v_b,v_e],options)`
 use + to combine graphics objects

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \text{vector}([1,2])$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \text{matrix}([[1,2],[3,4]])$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \det(\text{matrix}([[1,2],[3,4]]))$$

$$Av = A*v \quad A^{-1} = A^{-1} \quad A^t = A.\text{transpose}()$$

methods: `nrows()` `ncols()` `nullity()` `rank()` `trace()`...

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \text{vector}([1,2])$$

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$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \det(\text{matrix}([[1,2],[3,4]]))$$

$$Av = A*v \quad A^{-1} = A^{-1} \quad A^t = A.\text{transpose}()$$

methods: `nrows()` `ncols()` `nullity()` `rank()` `trace()`...

Sage のモジュールとパッケージ Sage modules and packages
from module_name import * (多くが既に読み込み済)
 例 `calculus coding combinat crypto functions games geometry graphs groups logic matrix numerical plot probability rings sets stats`
sage.module_name.all.<tab> export されたコマンドを表示
 Std packages: Maxima GP/PARI GAP Singular R Shell...
 Opt packages: Biopython Fricas(Axiom) Gnuplot Kash...
%package_name then use package command syntax
time command timing information を表示
from module_name import * (many preloaded)
 e.g. `calculus coding combinat crypto functions games geometry graphs groups logic matrix numerical plot probability rings sets stats`
sage.module_name.all.<tab> shows exported commands
 Std packages: Maxima GP/PARI GAP Singular R Shell...
 Opt packages: Biopython Fricas(Axiom) Gnuplot Kash...
%package_name then use package command syntax
time command to show timing information

離散数学 Discrete math

$\lfloor x \rfloor = \text{floor}(x)$ $\lceil x \rceil = \text{ceil}(x)$
 n を k で割った余り = $n\%k$ $k|n$ iff $n\%k==0$
 $n! = \text{factorial}(n)$ $\binom{x}{m} = \text{binomial}(x,m)$
 $\phi = \text{golden_ratio}$ $\phi(n) = \text{euler_phi}(n)$
 文字列: 例 `s = 'Hello' = "Hello" = ""+"He"+"llo"`
`s[0]='H' s[-1]='o' s[1:3]='el' s[3:]='lo'`
 リスト: 例 `[1,'Hello',x] = []+[1,'Hello']+[x]`
 タプル: 例 `(1,'Hello',x)` (immutable)
 集合: 例 `{1,2,1,a} = Set([1,2,1,'a']) (= {1,2,a})`
 集合の内包的記法 \approx リストの内包表記, 例
 $\{f(x) : x \in X, x > 0\} = \text{Set}([f(x) \text{ for } x \text{ in } X \text{ if } x > 0])$
 $\lfloor x \rfloor = \text{floor}(x)$ $\lceil x \rceil = \text{ceil}(x)$
 Remainder of n divided by $k = n\%k$ $k|n$ iff $n\%k==0$
 $n! = \text{factorial}(n)$ $\binom{x}{m} = \text{binomial}(x,m)$
 $\phi = \text{golden_ratio}$ $\phi(n) = \text{euler_phi}(n)$
 Strings: e.g. `s = 'Hello' = "Hello" = ""+"He"+"llo"`
`s[0]='H' s[-1]='o' s[1:3]='el' s[3:]='lo'`
 Lists: e.g. `[1,'Hello',x] = []+[1,'Hello']+[x]`
 Tuples: e.g. `(1,'Hello',x)` (immutable)
 Sets: e.g. `{1,2,1,a} = Set([1,2,1,'a']) (= {1,2,a})`
 List comprehension \approx set builder notation, e.g.
 $\{f(x) : x \in X, x > 0\} = \text{Set}([f(x) \text{ for } x \text{ in } X \text{ if } x > 0])$