

# Sage Quick Reference: Calculus

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<http://wiki.sagemath.org/quickref>

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## Builtin constants and functions

Constants:  $\pi = \text{pi}$   $e = \text{e}$   $i = \text{I} = \text{i}$

$\infty = \text{oo} = \text{infinity}$   $\text{NaN} = \text{NaN}$   $\log(2) = \text{log2}$

$\phi = \text{golden\_ratio}$   $\gamma = \text{euler\_gamma}$

$0.915 \approx \text{catalan}$   $2.685 \approx \text{khinchin}$

$0.660 \approx \text{twinprime}$   $0.261 \approx \text{merten}$   $1.902 \approx \text{brun}$

Approximate:  $\text{pi.n(digits=18)} = 3.14159265358979324$

Builtin functions:  $\sin$   $\cos$   $\tan$   $\sec$   $\csc$   $\cot$   $\sinh$   
 $\cosh$   $\tanh$   $\text{sech}$   $\text{csch}$   $\text{coth}$   $\log$   $\ln$   $\exp$  ...

## Defining symbolic expressions

Create symbolic variables:

`var("t u theta")` or `var("t,u,theta")`

Use  $*$  for multiplication and  $\wedge$  for exponentiation:

$2x^5 + \sqrt{2} = 2*x^5 + \text{sqrt}(2)$

Typeset: `show(2*theta^5 + sqrt(2))`  $\longrightarrow 2\theta^5 + \sqrt{2}$

## Symbolic functions

Symbolic function (can integrate, differentiate, etc.):

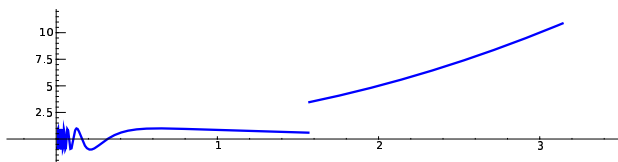
`f(a,b,theta) = a + b*theta^2`

Also, a "formal" function of theta:

`f = function('f',theta)`

Piecewise symbolic functions:

`Piecewise([(0,pi/2),sin(1/x)],[(pi/2,pi),x^2+1])`



## Python functions

Defining:

```
def f(a, b, theta=1):
    c = a + b*theta^2
    return c
```

Inline functions:

`f = lambda a, b, theta = 1: a + b*theta^2`

## Simplifying and expanding

Below  $f$  must be symbolic (so **not** a Python function):

Simplify: `f.simplify_exp()`, `f.simplify_full()`,  
`f.simplify_log()`, `f.simplify_radical()`,  
`f.simplify_rational()`, `f.simplify_trig()`

Expand: `f.expand()`, `f.expand_rational()`

## Equations

Relations:  $f = g: f == g$ ,  $f \neq g: f != g$ ,

$f \leq g: f <= g$ ,  $f \geq g: f >= g$ ,

$f < g: f < g$ ,  $f > g: f > g$

Solve  $f = g$ : `solve(f == g, x)`, and

`solve([f == 0, g == 0], x, y)`

`solve([x^2+y^2==1, (x-1)^2+y^2==1], x, y)`

Solutions:

`S = solve(x^2+x+1==0, x, solution_dict=True)`

`S[0][x]` `S[1][x]` are the solutions

Exact roots: `(x^3+2*x+1).roots(x)`

Real roots: `(x^3+2*x+1).roots(x,ring=RR)`

Complex roots: `(x^3+2*x+1).roots(x,ring=CC)`

## Factorization

Factored form: `(x^3-y^3).factor()`

List of (factor, exponent) pairs:

`(x^3-y^3).factor_list()`

## Limits

$\lim_{x \rightarrow a} f(x) = \text{limit}(f(x), x=a)$

$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = \text{limit}(\sin(x)/x, x=0)$

$\lim_{x \rightarrow a^+} f(x) = \text{limit}(f(x), x=a, \text{dir}='plus')$

`limit(1/x, x=0, dir='plus')`

$\lim_{x \rightarrow a^-} f(x) = \text{limit}(f(x), x=a, \text{dir}='minus')$

`limit(1/x, x=0, dir='minus')`

## Derivatives

$\frac{d}{dx}(f(x)) = \text{diff}(f(x), x) = f.\text{diff}(x)$

$\frac{\partial}{\partial x}(f(x, y)) = \text{diff}(f(x, y), x)$

`diff = differentiate = derivative`

`diff(x*y + sin(x^2) + e^(-x), x)`

## Integrals

$\int f(x)dx = \text{integrate}(f(x), x)$

`integral(f(x), x)`

$\int_a^b f(x)dx = \text{integrate}(f(x), x, a, b)$

`integral(f(x), x, a, b)`

$\int_a^b f(x)dx \approx \text{numerical\_integrate}(f(x), x, a, b)$

`numerical_integral(f(x), x, a, b)`

`assume(...):`

`assume(x>0)`

## Taylor and power series

Taylor polynomial: `taylor(f(x), x, a, n)`

`taylor(f(x), x, a, n)`

`taylor(sin(x), x, 0, 5)`

Partial fraction decomposition: `partial_fraction(f(x))`

`(x^2/(x+1))^3`

## Numerical root finding

Numerical root finding: `find_root(f(x), a, b)`

`(x^2 - 2)`

Maximize: `find_maximum(f(x), a, b)`

`f.find_maximum(x)`

Minimize: `find_minimum(f(x), a, b)`

`f.find_minimum(x)`

Minimization: `minimize(f(x), a, b)`

`minimize(x)`

## Multivariable Calculus

Gradient: `f.gradient(x, y)`

`(x^2+y^2)`

Hessian: `f.hessian(x, y)`

`(x^2+y^2)`

Jacobian matrix: `f.jacobian(x, y)`

`jacobian(x, y)`

## Summing and products

Sum: `sum(f(x), x, a, b)`

`(x^2+y^2)`

Product: `prod(f(x), x, a, b)`

`(x^2+y^2)`

Not yet implemented: `sum(1/x, x, a, b)`

`SR(sage.calculus.symbolic)`