Sage Quick Reference: Calculus

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Sage Version 3.4
http://wiki.sagemath.org/quickref

組込み定数と函数　Builtin constants and functions

定数: π=`pi`　e=`e`　i=`I`=`i`
　∞=`oo`=`infinity`　NaN=`NaN`　log(2)=`log2`
　ϕ=`golden_ratio`　γ=`euler_gamma`
　0.915≈`catalan`　2.685≈`khinchin`　0.660≈`twinprime`
　0.261≈`merten`　1.902≈`brun`

近似: `pi.n(digits=18)` = 3.14159265358979324

組込み函数: `sin cos tan sec csc cot sinh cosh tanh sech csch coth log ln exp` ...

Constants: π=`pi`　e=`e`　i=`I`=`i`
　∞=`oo`=`infinity`　NaN=`NaN`　log(2)=`log2`
　ϕ=`golden_ratio`　γ=`euler_gamma`
　0.915≈`catalan`　2.685≈`khinchin`　0.660≈`twinprime`
　0.261≈`merten`　1.902≈`brun`
Approximate: `pi.n(digits=18)` = 3.14159265358979324
Builtin functions: `sin cos tan sec csc cot sinh cosh tanh sech csch coth log ln exp` ...

シンボリックな数式の定義　Defining symbolic expressions

不定元 (symbolic variable) の生成:
　`var("t u theta")` or `var("t,u,theta")`

かけ算は `*`, 冪乗は `^`: $2x^5+\sqrt{2}$ =`2*x^5 + sqrt(2)`

タイプセット: `show(2*theta^5 + sqrt(2))` $\longrightarrow 2\theta^5+\sqrt{2}$

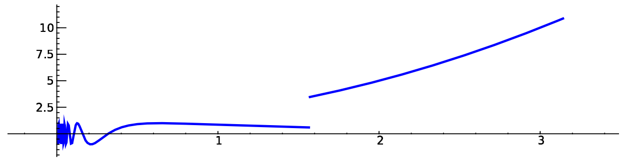
Create symbolic variables:
　`var("t u theta")` or `var("t,u,theta")`
Use `*` for multiplication and `^` for exponentiation:
　$2x^5+\sqrt{2}$ =`2*x^5 + sqrt(2)`
Typeset: `show(2*theta^5 + sqrt(2))` $\longrightarrow 2\theta^5+\sqrt{2}$

シンボリックな函数　Symbolic functions

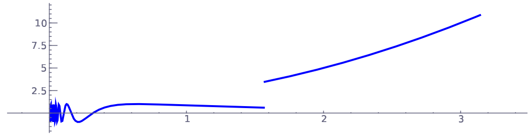
シンボリックな函数 (Symbolic function) (微分や積分ができる):
　`f(a,b,theta) = a + b*theta^2`

theta の "形式的な" 函数: `f = function('f',theta)`

区分的なシンボリックな函数:
`Piecewise([[(0,pi/2),sin(1/x)],[(pi/2,pi),x^2+1]])`



Symbolic function (can integrate, differentiate, etc.):
　`f(a,b,theta) = a + b*theta^2`
Also, a "formal" function of theta:
　`f = function('f',theta)`
Piecewise symbolic functions:
`Piecewise([[(0,pi/2),sin(1/x)],[(pi/2,pi),x^2+1]])`



Python の関数　Python functions

定義:
```
def f(a, b, theta=1):
    c = a + b*theta^2
    return c
```
インライン関数:
```
f = lambda a, b, theta = 1: a + b*theta^2
```
Defining:
```
def f(a, b, theta=1):
    c = a + b*theta^2
    return c
```
Inline functions:
```
f = lambda a, b, theta = 1: a + b*theta^2
```

簡単化と展開　Simplifying and expanding

以下の f は, シンボリックな函数でなければならない (Python の関数では**ない**):

簡単化: `f.simplify_exp()` `f.simplify_full()`
　`f.simplify_log()` `f.simplify_radical()`
　`f.simplify_rational()` `f.simplify_trig()`

展開: `f.expand()` `f.expand_rational()`

Below f must be symbolic (so **not** a Python function):
Simplify: `f.simplify_exp()` `f.simplify_full()`
　`f.simplify_log()` `f.simplify_radical()`
　`f.simplify_rational()` `f.simplify_trig()`
Expand: `f.expand()` `f.expand_rational()`

等式　Equations

関係式: $f=g$: `f == g`,　$f\neq g$: `f != g`,
　$f\le g$: `f <= g`,　$f\ge g$: `f >= g`,
　$f<g$: `f < g`,　$f>g$: `f > g`

$f=g$ を解く: `solve(f == g, x)` とか
　`solve([f == 0, g == 0], x,y)`
　`solve([x^2+y^2==1, (x-1)^2+y^2==1],x,y)`

解: `S = solve(x^2+x+1==0, x, solution_dict=True)`
　`S[0]["x"]` `S[1]["x"]` are the solutions

厳密解: `(x^3+2*x+1).roots(x)`
実数解: `(x^3+2*x+1).roots(x,ring=RR)`
複素数解: `(x^3+2*x+1).roots(x,ring=CC)`

Relations: $f=g$: `f == g`,　$f\neq g$: `f != g`,
　$f\le g$: `f <= g`,　$f\ge g$: `f >= g`,
　$f<g$: `f < g`,　$f>g$: `f > g`
Solve $f=g$: `solve(f == g, x)`, and
　`solve([f == 0, g == 0], x,y)`
　`solve([x^2+y^2==1, (x-1)^2+y^2==1],x,y)`
Solutions:
　`S = solve(x^2+x+1==0, x, solution_dict=True)`
　`S[0]["x"]` `S[1]["x"]` are the solutions
Exact roots: `(x^3+2*x+1).roots(x)`
Real roots: `(x^3+2*x+1).roots(x,ring=RR)`
Complex roots: `(x^3+2*x+1).roots(x,ring=CC)`

因数分解　Factorization

因数分解: `(x^3-y^3).factor()`
(因数, 巾) というペアのリスト: `(x^3-y^3).factor_list()`

Factored form: `(x^3-y^3).factor()`
List of (factor, exponent) pairs: `(x^3-y^3).factor_list()`

極限　Limits

$\lim_{x\to a} f(x)$ = `limit(f(x), x=a)`
　`limit(sin(x)/x, x=0)`
$\lim_{x\to a^+} f(x)$ = `limit(f(x), x=a, dir='plus')`
　`limit(1/x, x=0, dir='plus')`
$\lim_{x\to a^-} f(x)$ = `limit(f(x), x=a, dir='minus')`
　`limit(1/x, x=0, dir='minus')`

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$\lim_{x\to a^-} f(x)$ = `limit(f(x), x=a, dir='minus')`
　`limit(1/x, x=0, dir='minus')`

微分　Derivatives

$\frac{d}{dx}(f(x))$ = `diff(f(x),x)` = `f.diff(x)`
$\frac{\partial}{\partial x}(f(x,y))$ = `diff(f(x,y),x)`
`diff` = `differentiate` = `derivative`
　`diff(x*y + sin(x^2) + e^(-x), x)`

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$\frac{\partial}{\partial x}(f(x,y))$ = `diff(f(x,y),x)`
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積分　Integrals

$\int f(x)dx$ = `integral(f,x)` = `f.integrate(x)`
　`integral(x*cos(x^2), x)`
$\int_a^b f(x)dx$ = `integral(f,x,a,b)`
　`integral(x*cos(x^2), x, 0, sqrt(pi))`
$\int_a^b f(x)dx \approx$ `numerical_integral(f(x),a,b)[0]`

`numerical_integral(x*cos(x^2),0,1)[0]`

`assume(...)`: 積分の際に質問されたら使う.

`assume(x>0)`

$\int f(x)dx =$ `integral(f,x)` = `f.integrate(x)`

`integral(x*cos(x^2), x)`

$\int_a^b f(x)dx =$ `integral(f,x,a,b)`

`integral(x*cos(x^2), x, 0, sqrt(pi))`

$\int_a^b f(x)dx \approx$ `numerical_integral(f(x),a,b)[0]`

`numerical_integral(x*cos(x^2),0,1)[0]`

`assume(...)`: use if integration asks a question

`assume(x>0)`

テイラー展開と部分分数展開 Taylor and partial fraction expansion

a に関する次数 n のテイラー多項式:

`taylor(f,x,a,n)`$\approx c_0 + c_1(x-a) + \cdots + c_n(x-a)^n$

`taylor(sqrt(x+1), x, 0, 5)`

部分分数展開: `(x^2/(x+1)^3).partial_fraction()`

Taylor polynomial, deg n about a:

`taylor(f,x,a,n)`$\approx c_0 + c_1(x-a) + \cdots + c_n(x-a)^n$

`taylor(sqrt(x+1), x, 0, 5)`

Partial fraction: `(x^2/(x+1)^3).partial_fraction()`

数値解と最適化 Numerical roots and optimization

数値解: `f.find_root(a, b, x)`

`(x^2 - 2).find_root(1,2,x)`

最大化: $f(x_0) = m$ が極大となる (m, x_0) を探す

`f.find_maximum_on_interval(a, b, x)`

最小化: $f(x_0) = m$ が極小となる (m, x_0) を探す

`f.find_minimum_on_interval(a, b, x)`

最小化: `minimize(f,`*start_point*`)`

`minimize(x^2+x*y^3+(1-z)^2-1, [1,1,1])`

Numerical root: `f.find_root(a, b, x)`

`(x^2 - 2).find_root(1,2,x)`

Maximize: find (m, x_0) with $f(x_0) = m$ maximal

`f.find_maximum_on_interval(a, b, x)`

Minimize: find (m, x_0) with $f(x_0) = m$ minimal

`f.find_minimum_on_interval(a, b, x)`

Minimization: `minimize(f,`*start_point*`)`

`minimize(x^2+x*y^3+(1-z)^2-1, [1,1,1])`

多変数函数 Multivariable calculus

勾配 (Gradient): `f.gradient()` or `f.gradient(`*vars*`)`

`(x^2+y^2).gradient([x,y])`

ヘッセ行列 (Hessian): `f.hessian()`

`(x^2+y^2).hessian()`

ヤコビ行列: `jacobian(f,`*vars*`)`

`jacobian(x^2 - 2*x*y, (x,y))`

Gradient: `f.gradient()` or `f.gradient(`*vars*`)`

`(x^2+y^2).gradient([x,y])`

Hessian: `f.hessian()`

`(x^2+y^2).hessian()`

Jacobian matrix: `jacobian(f,`*vars*`)`

`jacobian(x^2 - 2*x*y, (x,y))`

無限級数 Summing infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

まだ実装されていないが, Maxima を使うことが出来る:

`s = 'sum (1/n^2,n,1,inf), simpsum'`

`SR(sage.calculus.calculus.maxima(s))` $\longrightarrow \pi^2/6$

Not yet implemented, but you can use Maxima:

`s = 'sum (1/n^2,n,1,inf), simpsum'`

`SR(sage.calculus.calculus.maxima(s))` $\longrightarrow \pi^2/6$